Lack of universality in two-dimensional multicomponent spreading phenomena

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Magnetic entities are introduced in a two-dimensional Eden model as additional degrees of freedom in order to model multidomain spreading phenomena. The elements of the growth are "spins" taking q states and are coupled or not via an energy J, as in the Potts model, thus leading to a competition between species. The internal cluster spreading is found to result from a competition between growing domains. Complex mechanisms such as trapping, jamming, and coalescence occur between the growing domains. A large variety of critical and nonuniversal regimes, from subcritical to self-organized critical behaviors, are obtained depending on nonuniversal parameters such as the lattice structure, the number of internal degrees of freedom q, and the coupling J. For the square lattice, the fractal dimension is 1.50 and the mass distribution exponent τ is 1.63. For the triangular lattice, the fractal dimension varies from 1.70 to 1.83 depending on the coupling value and the mass distribution exponent τ also varies from 1.67 to 1.98 depending on the coupling value. The correspondence and differences with respect to the percolation phenomenon are outlined.

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I. INTRODUCTION

Spreading phenomena are common nonequilibrium processes in nature: surface wetting, viscous fingering, liquid invasion in porous medium, grain coalescence in alloys, fracture propagation, but also evolution of territories or population of insect swarms, virus propagation, and so on. For spreading processes driven by cooperative or nonlinear evolution rules, the systems develop patterns which often reach a high level of complexity [1,2]. In modern statistical physics, the understanding of a wide variety of natural spreading phenomena is approached by inventing simple models. Their complexity is studied in terms of "criticality" [2]. A spreading phenomenon is said to be in a critical regime when the characteristic length of the pattern becomes infinite as the spatial extension of the spreading tends to infinity, i.e., in a fractal growth regime [1]. A subcritical regime occurs when the spreading stops leading to a finite characteristic length of the patterns. A supercritical regime will be said to take place when an infinite cluster can grow but inner patterns have a finite characteristic length, i.e., for nonscale invariant structures. The signatures of strict criticality are known to be, e.g., the fractality of a pattern or the power law behavior of the size distribution of spreading events [3].

The most simple nonequilibrium spreading model is the Eden model which describes the aggregation of identical particles. The model was imagined in order to mimic the growth of bacterian cells colonies [4] and was rapidly generalized to simulate other one-component spreading phenomena [5–9]. In the simplest version of the model [5], a single step of the growth consists in randomly selecting a particle on the surface of a seed, a cluster thereafter, and at random filling one of its empty neighbors by a new particle. The generated clusters are found to fill the entire available space showing trivial nonfractal compact structures.

However, in natural systems, the growing entities usually present additional degrees of freedom. Examples of multicomponent systems are alloys, fluids, magnets, ceramics, polymers, bacterian cells, viruses ,. . . .

In this paper, we consider an arbitrary number q of internal degrees of freedom for a simple growth model whence much extending the domain of applications of the Eden growth process. The model is defined in Sec. II. In Sec. III, we present the various critical regimes which can be found in this multicomponent Eden-Potts two-dimensional model. We observe the lack of universality of two-dimensional multicomponent spreading phenomena on different lattices. This is discussed in Sec. IV. A conclusion is drawn in Sec. V.

II. THE MULTICOMPONENT EDEN MODEL

In the multicomponent Eden model, the elements of the growth are represented by scalar "spins" σ_i taking q states and coupled by a dimensionless energy taking two values J and 0 as in the Potts model [10,11]. Starting from a single spin of state $\sigma_0=1$ as seed, the growth consists in successively selecting at random one site i, then the spin of state $1\le\sigma_i\le q$, on the cluster surface. From this site i, a new spin is glued on an empty nearest neighboring site j chosen at random. The state of this new spin is $1\le\sigma_j\le q$ and the value chosen with a probability

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$$\frac{\exp(J\delta_{\sigma_i\sigma_j})}{\exp(J)+q-1} \tag{1}$$

among the q states and while $\delta_{x,y}$ is the Kronecker function. This rule is repeated until a desired total number N of spins is reached. After being glued, each spin is *frozen forever.* This is quite different from the classical Potts model which is usually studied in its equilibrium state. The process of the present model is history dependent (non Markovian) and fully irreversible (far from equilibrium). The dynamics of the model are thus expected to be different from the classical Potts dynamics [11]. The parameter J can be related to the affinity of aggregation between different species of particles, or more generally J represents the "intensity" of the internal competition occurring between the q species. Instead of the dimensionless energy $J\in\,]-\infty,+\infty[$, it is better to consider a parameter p as the underlying control parameter

$$p=\frac{\exp(J)}{\exp(J)+q-1}\ . \tag{2}$$

One should note that for $p=1$, the Eden growth case is recovered because the seed species totally fills the growing cluster. A binary Eden growth occurs for $q=2$ [12]. For $q>1$ and for values of p strictly different from one, the growth rule leads to the existence of growing domains distributed in clusters. A domain is defined as a set of connected spins in the same state. Figures 1(a)–1(c) show three typical q-component clusters of $N=10\,000$ spins each for, respectively, $q=2$, 3, and 4 on the square lattice and for a different p value. It will be understood below that the p values chosen for this illustration correspond to a seed species domain which is very large and extends to the $N=10\,000$ cluster boundary when $J>0$. For $J<0$ (or $p<1/q$), antiferromagneticlike configurations are observed as shown in Fig. 1(d) for $q=3$ and $J=-1$. In this case, very small domains are seen. Each "color" in Figs. 1(a)–1(d) represents a spin species. While the overall cluster growth is strictly equivalent to the classical Eden process leading to round and compact clusters, the internal (colored) patterns show trapped or meandering domains of the various q species.

It is clear that each domain represents a hindrance for the growth of the other neighboring domains. Some

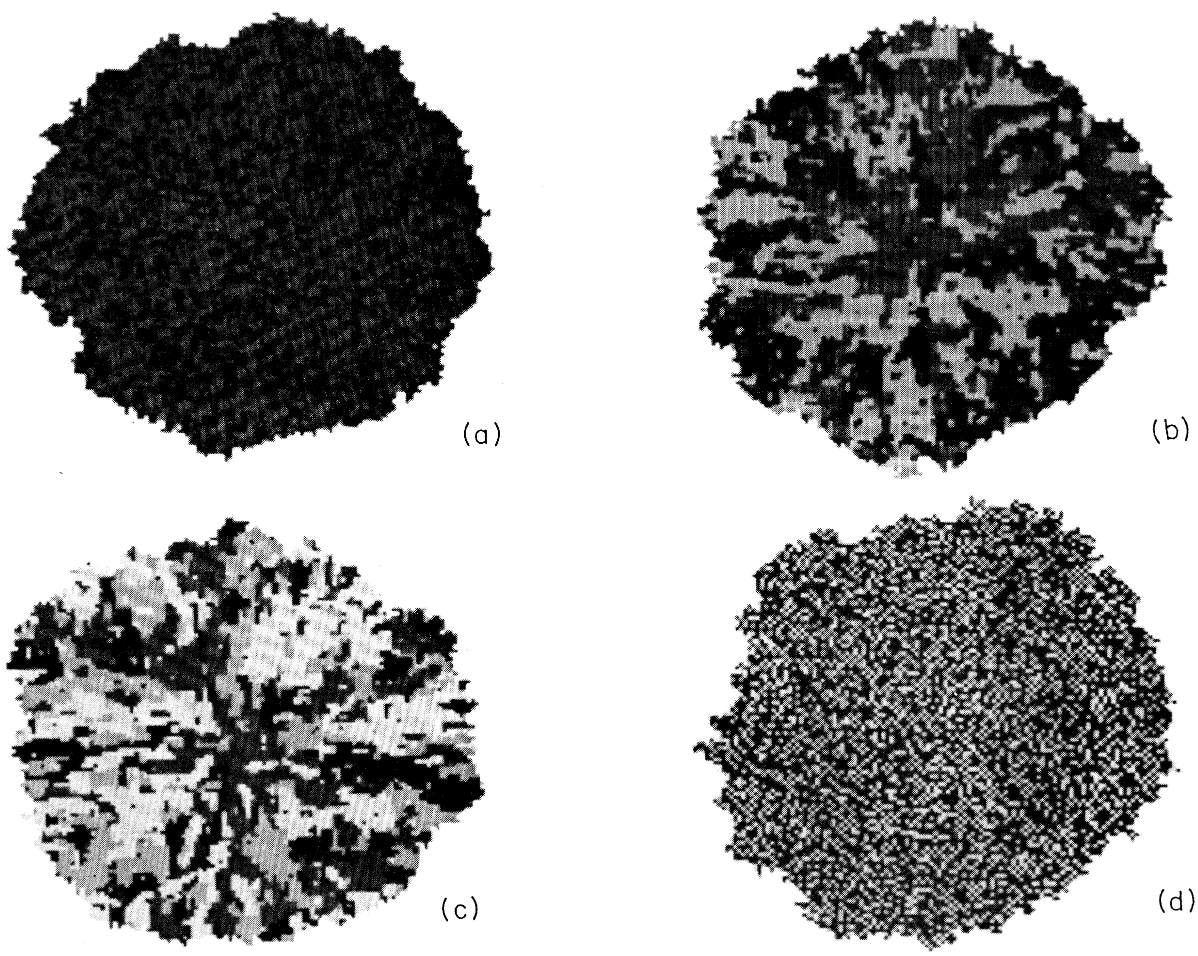

FIG. 1. Three typical Eden-Potts spin clusters of mass $N=10\,000$. (a) for $q=2$ and $p=0.75$; (b) for $q=3$ and $p=0.88$; (c) for $q=4$ and $p=0.92$; (d) for $q=3$ and $p=0.15$.

domains can be trapped inside large ones of other species or jammed between others. Domains of the same species can also coalesce during the growth. Such an observation raises the question of a percolationlike mechanism for the seed species domain and whether a critical p_c exists. Even though the growth rule of the model is very simple, it should thus generate complex kinetic phenomena. It can be emphasized that the growth is thus more complex than classical epidemic [9] or forest fire models [13] where only "*immune*," i.e., nongrowing sites, constitute obstacles for the spreading of a single component domain.

III. NUMERICAL RESULTS

A. The earliest stages of the cluster growth

At the very earliest stages of the cluster growth, the seed type state dominates the multicomponent spreading process. After some steps, new domains of the $(q-1)$ other species are "nucleated" and infect the seed cluster surface. The latter new domains also grow and the spreading phenomenon reaches a *steady state* in which domains of the q species *spread and compete* with each other on the surface of the growing cluster.

In such a steady state, the concentrations c_1 to c_q of the q kinds of species are expected to be $1/q$. Figure 2 presents the evolution of the concentration of the various species as a function of the cluster mass N for a $q=3$ multicomponent spreading phenomenon and for a positive coupling $J=1.5$. The concentration of the seed species c_1 decreases from 1.0 for $N=1$ and reaches $1/q$ for large N values. The other concentrations c_2 to c_q increase from 0 for $N=1$ and reach $1/q$ for large N values. For large N values, the concentrations c_1 to c_q are submitted to fluctuations around the steady-state value $1/q$.

Figure 3 presents the seed domain pertaining to a three-component cluster of $N=50\,000$ spins. The perimeter of this domain is very jagged reflecting the presence of many obstacles. It is of interest to know the conditions for which the spreading domains can grow forever or not. This is in line with classical percolation studies even though the present model is *a priori* very different. In percolation studies, the system is in a static state for which both geometrical and physical properties are searched for at fixed concentrations. Here, the system is in a dynamical steady state with roughly constant concentrations at large Monte Carlo times.

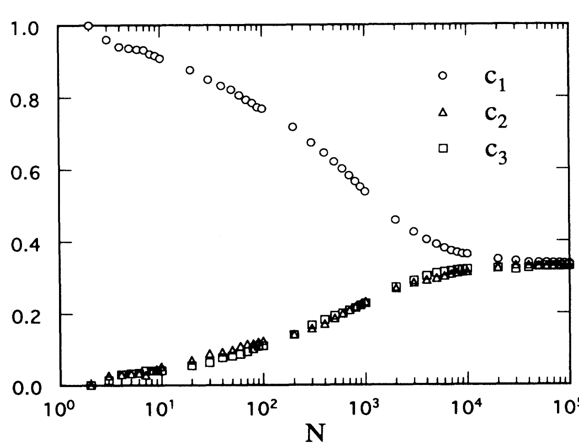

FIG. 2. The evolution of the concentrations of the various components in $q=3$ Eden-Potts clusters as a function of the mass N. The coupling constant J was set to 1.5. Each dot represents an average of the concentration over 25 clusters.

FIG. 3. The seed domain of a $N=50\,000$ Eden-Potts cluster of $q=3$ components. The coupling constant J was set to 1.5.

We have, e.g., numerically performed the measure of the probability P_∞ to have the seed type domain reaching the surface, i.e., when the seed type of spin is always part of the growth front and connected to the seed by similar spins. We have investigated large clusters for various q and J, thus p values on several two-dimensional lattices. Clusters of about one million spins have been simulated. The mass distribution of the various domains has also been measured. Such a numerical study will help us to discern the possible links between equilibrium spin models or random percolation and the present multicomponent spreading model.

B. The honeycomb lattice

For the honeycomb lattice (having a coordination number $z=3$), the probability P_∞ is numerically found to be zero for all $0 \le p < 1$ values for clusters with mass N up to 8×10^5. For p values very close to 1, P_∞ is found to be nonzero but P_∞ shrinks to zero as the size of the clusters is increased. The multidomain spreading is always found to be in a *subcritical regime* for any q and $0 \le p < 1$ values.

C. The square lattice

Figure 4 presents the percolation probability P_∞ as a function of p for two-component ($q=2$) clusters of size $N=2\times10^5$ on a square lattice. The probability $P_\infty(p)$ jumps from zero to unity near some critical value $p_c\approx0.78$. Each dot represents an average of P_∞ over 40 two-component clusters. Figure 5 presents the threshold value p_c as a function of the mass N of the clusters. Each dot of Fig. 5 represents the simulation of about 1000 clusters. The value of $p_c(N)$ was found to reach asymptotically the value $p_c=0.83\pm0.03$ for $N\leq8\times10^5$, but does not seem to ever reach unity. Finite-size scaling arguments of classical percolation [14] equate the connectivity length $\xi\sim(p_c-p)^{-\nu}$ with the lattice or cluster size $N^{1/2}$ resulting in the relation

$$p_c(\infty)-p_c(N)\sim N^{-1/2\nu}\ , \qquad (3)$$

which gives a direct measure of the critical exponent ν. This theoretical law is drawn in Fig. 5. The critical exponent is found to be $\nu=0.81\pm0.06$. Thus the transition at $p_c=0.83\pm0.03$ ($J_c=1.58\pm0.22$) does not seem to be *a finite-size artifact.*

In this particular $q=2$ case, the domains of both species are only finite below p_c, and infinite domains can grow in the cluster with a probability [14]

$$P_\infty\sim(p-p_c)^\beta \qquad (4)$$

above p_c. The critical exponent β is numerically found to be $\beta=0.7\pm0.1$ from the data in Fig. 4. Because the multidomain competition cannot have any winner or conqueror in the steady state, infinite domains of both species can coexist into an infinite cluster.

For $q>2$, $P_\infty(p)$ was numerically found to converge to zero for large N values and for all $0\leq p<1$ values. The latter result means that *only finite domains are found for $q>2$ on a square lattice.* For $q>2$, the multidomain spreading phenomenon is *subcritical.* This was verified with clusters up to 8×10^5 spins on the square lattice.

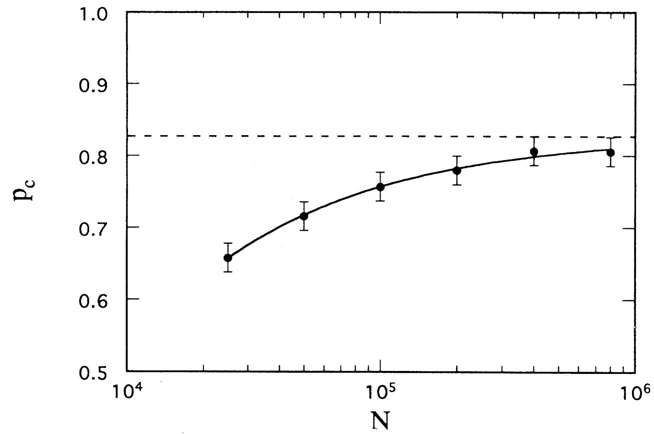

FIG. 5. The threshold value p_c as a function of the size N of the clusters. The continuous line is the fittest power law decay of $p_c(N)$ towards the asymptotic threshold $p_c(\infty)=0.83$ [see Eq. (2)]. The dashed horizontal line cuts the vertical axis at the latter threshold.

In terms of a polychromatic invasion problem, a *panchromatic* regime [15] can thus occur for $p>p_c$ only for the $q=2$ case on the square lattice. This $q=2$ case is therefore the only one investigated for the mass distribution behavior and the fractal dimensionality of the cluster. The mass-distribution $n(s)$ of the domains obtained by the Hosen-Kopelman numerical method [16] is assumed to follow the scaling relation

$$n(s)\sim s^{-\tau}\exp(-s/s_\xi)\ , \qquad (5)$$

where s_ξ is the characteristic mass of the domain and diverges in the critical situations [3]. For the square lattice, the strict power-law (for $s_\xi\rightarrow+\infty$) behavior characterizing the *critical* behavior is found only at p_c with the exponent τ numerically estimated to be $\tau=1.63\pm0.05$ (see Fig. 6 where the curve was obtained from an average over

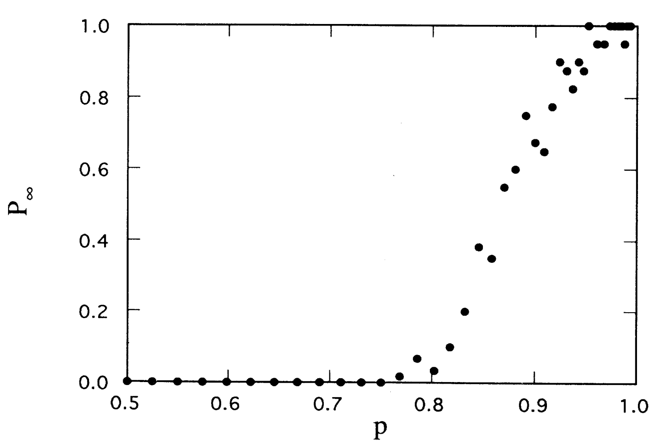

FIG. 4. The probability P_∞ to have an infinite domain grown from the seed as a function of p and for $q=2$ on a two-dimensional square lattice. The curve is drawn for $N=200\,000$ spins.

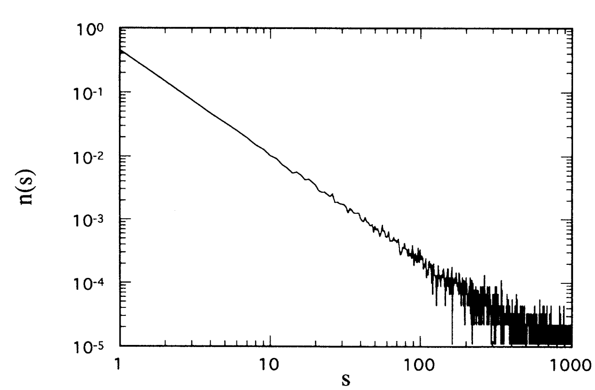

FIG. 6. The mass distribution of the domains into two-component clusters of 200 000 spins on a square lattice for $p=0.83\approx p_c$.

40 clusters of mass $N=5\times10^5$ for $p\approx p_c$). The deviations from the power-law behavior for large mass values are occurring from well known finite-size effects [14].

Figure 7 shows a spanning seed cluster at p_c for a two-component cluster of mass $N=100\,000$. The distribution of the various components is homogeneous (in large clusters). The fractal dimension of each component is thus $D_f=2.0$ as estimated by the gyration radius method [17]. However, the seed domains which span the large clusters are found to be fractal at p_c, a property which is another signature of criticality [1,18,19]. The fractal dimension D_f of the seed domain at p_c is numerically found to be $D_f=1.5\pm0.1$ which is lower than the fractal dimension $D_f=91/48$ (≈1.86) of classical percolating clusters [14,19]. One should also note that the hyperscaling relation [14] of classical percolation

$$D_f(\tau-1)=d\ , \tag{6}$$

which relates the fractal dimension D_f, the size-distribution exponent τ, and the lattice dimension d (herein $d=2$) does not hold here.

Summarizing what happens for $q=2$ on the square lattice ($z=4$), the spreading regime is *critical* at p_c, *subcritical* below p_c because the domains are only finite, and above p_c, the regime is *supercritical* because P_∞ is different from zero and $n(s)$ does not follow a power-law behavior with a nonclassical exponent.

D. The triangular lattice

For the triangular lattice ($z=6$), the spreading process is quite different. P_∞ values are finite for all values $0\le p<1$. Thus the $q=2$ steady-state case is *critical* whatever the value of the external parameter $0\le p<1$. The mass-distribution $n(s)$ for the special case of a two-component ($q=2$) spreading process, in particular, for the $p=\frac{1}{2}$ case is shown in Fig. 8. The curve is a power law with an exponent $\tau=1.96\pm0.08$, a value which is close to the two-dimensional percolation value $\tau=187/91$ (≈2.05). The same type of power-law behavior is found for all values $0\le p<1$. Figure 9 presents the estimated values of τ as a function of the parameter p. Each value is an average over 40 clusters of mass $N=5\times10^5$. The critical exponent τ seems to be dependent of p. The change of the τ value is well marked for p close to 1.

Moreover, the fractal dimension of the seed domain is found to be $D_f=1.83\pm0.05$ for $p=\frac{1}{2}$. This value is close to the fractal dimension (91/48) of classical percolating two-dimensional clusters. Figure 9 presents the estimated values of τ as a function of the parameter p. The critical exponent D_f seems to be weakly dependent of p except for p close to one where D_f falls to 1.7. Close to $p=1$, the hyperscaling relation of Eq. (6) does not hold for a two-component spreading on the triangular lattice. Further extensive simulations should make the values of the critical exponents more precise and clarify the p dependence on D_f and τ. A spanning seed domain in a large cluster $N=100\,000$ is shown in Fig. 10.

For $q>2$ and for any $0\le p<1$ values, $P_\infty(p)$ converges towards zero for large N values and only finite domains grow. This was verified for clusters of mass up to $N=8\times10^5$.

IV. DISCUSSION

The critical regimes for the various lattice cases examined here are summarized in Table I. In view of the above features of the spreading and in line with statistical mechanics studies, it is natural to relate the findings to percolation models. The major difference between the

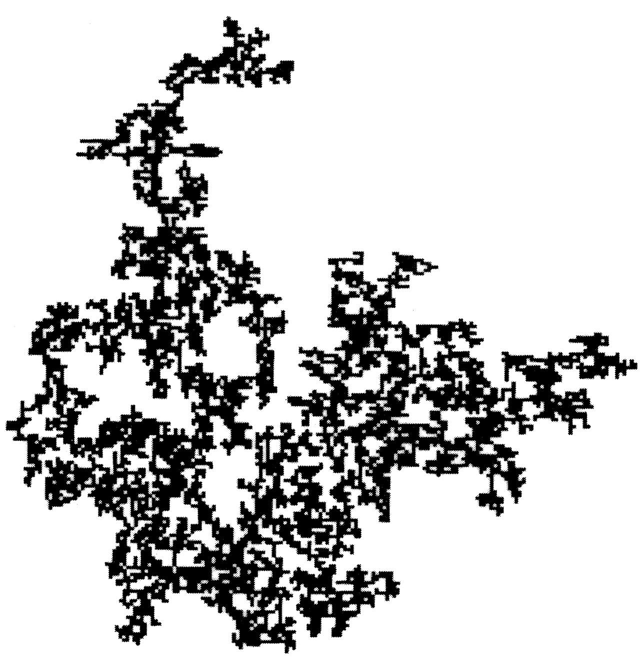

FIG. 7. A spanning seed domain (at p_c) in a cluster of mass $N=100\,000$ on the square lattice.

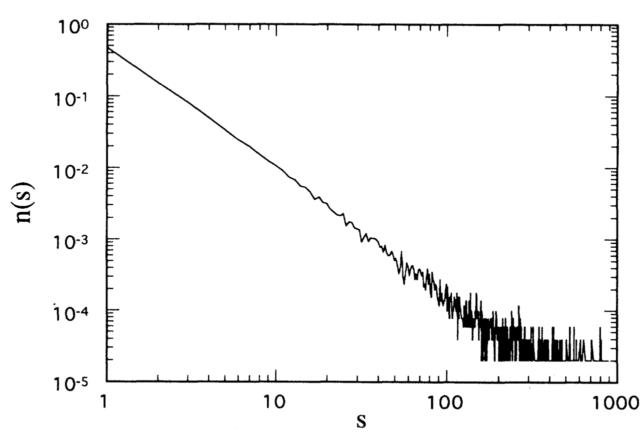

FIG. 8. The mass distribution of the domains into two-component clusters of 200 000 spins on a triangular lattice for $p=\frac{1}{2}$.

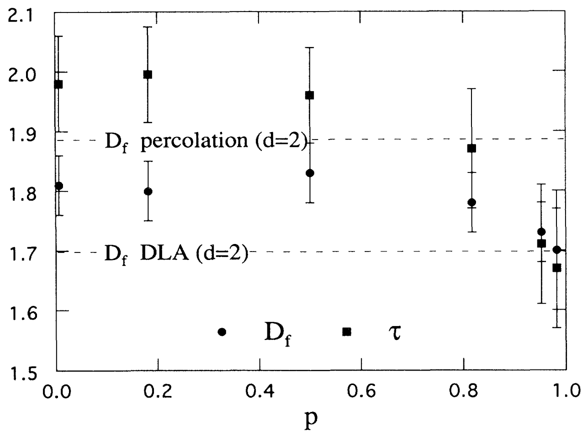

FIG. 9. The critical exponents D_f and τ as a function of p for the Eden-Potts model on a triangular lattice. Each data point represents an average over 40 clusters of mass $N=5\times10^5$.

above growth model and percolation is that the latter is characterized by a static disorder while growth induces herein some order (or correlations) via the coupling parameter. Unconstrained percolation [14] is, however, recovered for the particular case of decoupled spins, i.e., for $J=0$ (or $p=1/q$). Let us examine the different cases.

For unconstrained site percolation on a honeycomb lattice, the critical concentration is $c_s \approx 0.70$ [14,19] which is quite larger than the steady-state concentration $1/q$ of the multicomponent spreading. Percolation is thus not possible for $p=1/q$. For $p<1/q$, the proximity of spins in the same state is less favorable than for the $p=1/q$ case, and percolation is expected to not occur. For $p>1/q$, correlations seems to be not sufficient to induce percolation.

For unconstrained site percolation on the square lattice, the critical concentration is $c_s \approx 0.593$. Since $c_s > \frac{1}{2}$, this does not allow for a two equivalent component simultaneous percolation. This is thus in contrast to the

FIG. 10. A spanning seed domain (at $p=0$) in a cluster of mass $N=100\,000$ on the triangular lattice.

TABLE I. Critical regimes of the multicomponent Eden-Potts model for various two-dimensional lattices and for various values of q.

Lattice	z	q	Criticality
honeycomb	3	>1	subcritical
square	4	2	critical at $p_c \approx 0.83 \pm 0.03$
square	4	>2	subcritical
triangular	6	2	self-organized
triangular	6	>2	subcritical

present two-component Eden model on the square lattice for which a critical state exists. For $p<1/q$, the proximity of spins in the same state is less favorable than for the $p=1/q$ case, and percolation should not occur. For $p>1/q$, large domains can be observed, of course. The size of these domains increases with p. On length scales larger than the domain size, the coalescence of domains occurs. Thus, one expects to recover percolationlike phenomenon on large length scales for the present growth model [20]. However, such a percolation could not correspond to classical percolation on a square lattice since the domains do not conserve the symmetry of the underlying lattice [20].

Nevertheless, the critical exponents of the two-component spreading on a square lattice are quite different from unconstrained percolation exponents (see Table II). This expresses that the criticality of the present model has a different nature from that of percolation.

For unconstrained site percolation on a triangular lattice, two infinite clusters can simultaneously exist, in fact, since $c_s=0.5$ [14]. In this case, the percolation threshold corresponds exactly to the steady state concentration $1/q \equiv \frac{1}{2}$ in the presently examined two-component Eden-Potts model. The growth is thus always driven (see Sec. III A) towards a steady state corresponding to the critical state of random site percolation. Such a behavior is similar to a self-organized critical phenomenon [21,22] where the order parameter of a far from equilibrium dynamical process is tuned to a value which corresponds to a critical point.

For p different from $1/q$, the order introduced via J could not destroy the percolation which is a disorder property. Moreover, trapping and jamming processes are not present in the unconstrained and static percolation model. Table II gives the critical exponents for the present model and percolation. The critical exponents found herein are, however, quite different from the classical percolation ones.

V. CONCLUSION

We have introduced a kinetic "Eden-Potts" growth model simulating a multicomponent spreading phenomenon, like a polychromatic invasion problem. The simple growth rule of the model already induces complex dynamical mechanisms like trapping, jamming, and coalescence of growing domains during the cluster

TABLE II. Critical exponents for, respectively, the two-dimensional multicomponent Eden-Potts model on square and triangular lattices, two-dimensional random percolation, and the two-dimensional $q=2$ Potts model. The exponent D_f is the fractal dimension of the spanning domains, τ characterizes the critical domain size distribution, ν is the correlation length exponent, and β is the order parameter exponent.

Critical exponent	$q=2$ Eden-Potts model Square lattice for $J=J_c$	Triangular lattice for $J=0$	Unconstrained percolation	$q=2$ Potts model
D_f	1.50 ± 0.10	1.83 ± 0.05	91/48	15/8
τ	1.63 ± 0.05	1.96 ± 0.08	187/91	31/15
ν	0.81 ± 0.06		4/3	1
β	0.70 ± 0.10		5/36	1/8

growth history. For the multicomponent spreading, various critical regimes are emphasized depending on the lattice structure, the number of internal degrees of freedom q, and the coupling J. Critical behaviors with infinite fractal domain growth are numerically found to occur only for the two-component spreading on square or triangular lattices. We should recall that the p regimes are also different.

The values of the critical exponents of the two-dimensional multicomponent Eden model are *nonuniversal,* i.e., they depend on the underlying two-dimensional lattice. The critical exponents which specify the universality class of the domain growth kinetics [23] are quite particular: their values are far from the traditional values of critical [24–27] and percolation [14,19] phenomena. Let us also add here that the fractal dimension differs from the cluster-cluster aggregation model [28] where $D_f=1.38$. They are also markedly distinguished from the Potts model exponents (see Table II).

The multicomponent Eden-Potts model is a nonequilibrium dynamical process indeed by opposition to the classical Potts model which is an at-equilibrium model. Quite interestingly, the present model leads to a nonequilibrium steady state where no species can dominate the others for all values of J (or p). This is in contrast to the Potts model in which a species can dominate the others above some critical $J_c=\ln(1+\sqrt{q})$ value [11]. The nature of the transition in both models is thus quite different.

The above model opens new domains of investigations in statistical physics. A more general theory than classical percolation, or equilibrium phase transition is needed to explain such nonuniversal behaviors of multicomponent spreading phenomena. Besides future theoretical work, further extensive simulations should be made in order to precise the critical exponent values. The nonuniversality regimes should be investigated on other lattices and with more general interactions.

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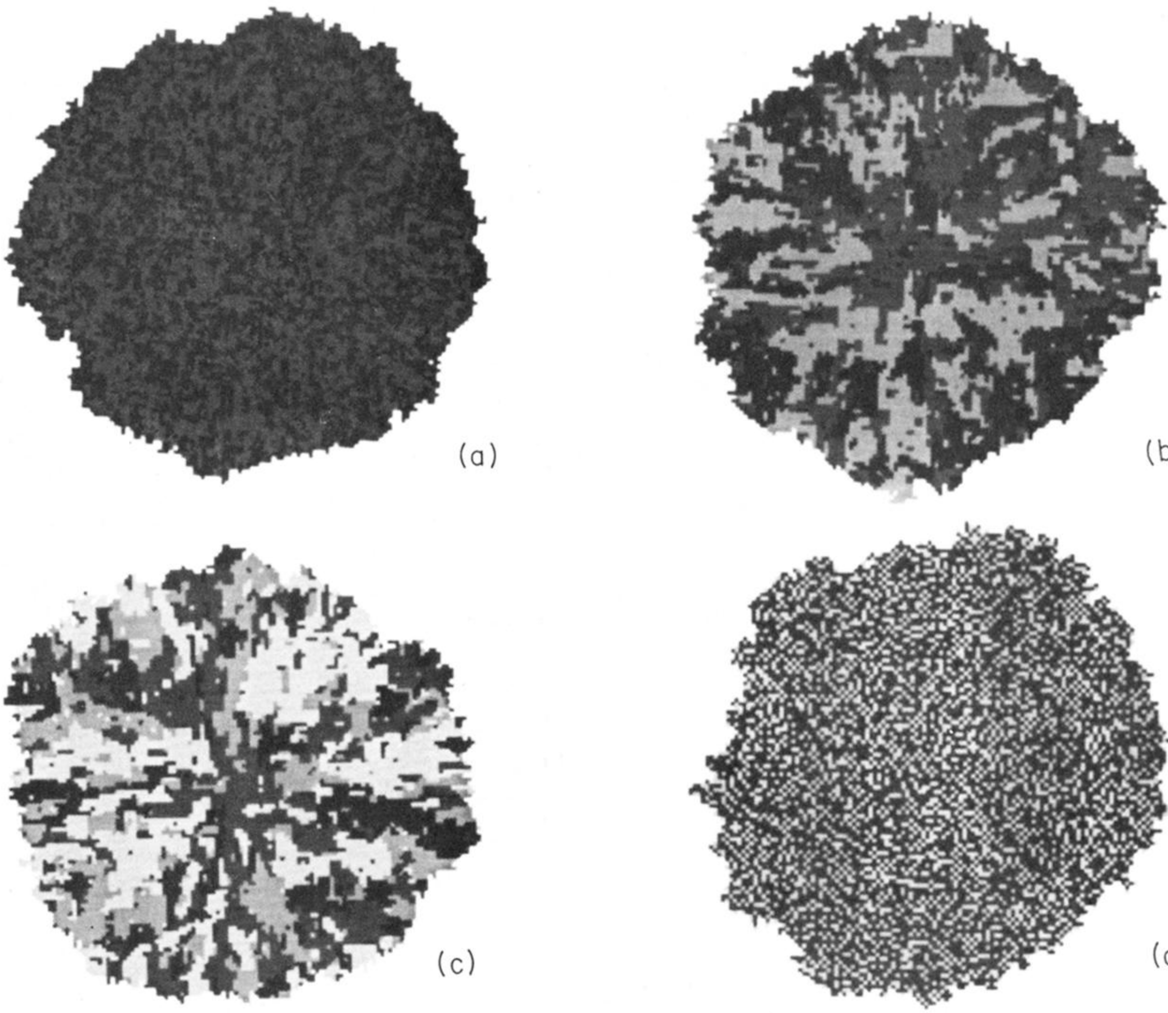

FIG. 1. Three typical Eden-Potts spin clusters of mass $N = 10\,000$. (a) for $q = 2$ and $p = 0.75$; (b) for $q = 3$ and $p = 0.88$; (c) for $q = 4$ and $p = 0.92$; (d) for $q = 3$ and $p = 0.15$.